Problem 26

In a tug-of-war game on one campus, 15 students pull on a rope at both ends in an effort to displace the central knot to one side or the other. Two students pull with force 196 N each to the right, four students pull with force 98 N each to the left, five students pull with force 62 N each to the left, three students pull with force 150 N each to the right, and one student pulls with force 250 N to the left. Assuming the positive direction to the right, express the net pull on the knot in terms of the unit vector. How big is the net pull on the knot? In what direction?

Solution

Draw a schematic of the knot and the forces acting on it. Let the horizontal distance be x and be positive going to the right. Then the unit vector in this direction is $\hat{\mathbf{x}}$.



The net force \mathbf{F}_{net} is the sum of the individual force vectors.

$$\begin{aligned} \mathbf{F}_{\text{net}} &= \sum_{i} \mathbf{F}_{i} \\ &= (-98 \text{ N})\mathbf{\hat{x}} + (-98 \text{ N})\mathbf{\hat{x}} + (-98 \text{ N})\mathbf{\hat{x}} + (-250 \text{ N})\mathbf{\hat{x}} \\ &+ (-62 \text{ N})\mathbf{\hat{x}} + (-62 \text{ N})\mathbf{\hat{x}} + (-62 \text{ N})\mathbf{\hat{x}} + (-62 \text{ N})\mathbf{\hat{x}} \\ &+ (196 \text{ N})\mathbf{\hat{x}} + (196 \text{ N})\mathbf{\hat{x}} + (150 \text{ N})\mathbf{\hat{x}} + (150 \text{ N})\mathbf{\hat{x}} \\ &= (-98 - 98 - 98 - 98 - 250 - 62 - 62 - 62 - 62 - 62 + 196 + 196 + 150 + 150 + 150) \text{ N} \mathbf{\hat{x}} \\ &= -110 \text{ N} \mathbf{\hat{x}} \end{aligned}$$

Take the magnitude of this net force vector to find how big the net pull is on the knot.

$$F_{\rm net} = |\mathbf{F}_{\rm net}| = \sqrt{(-110 \text{ N})^2} = 110 \text{ N}$$

Since the net force is negative, the knot is pulled to the left.